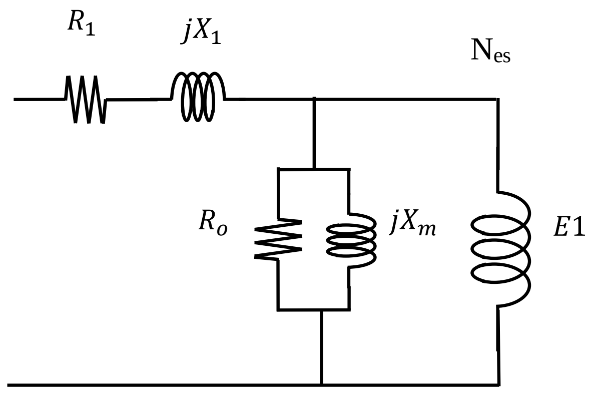
Equivalent Circuit of Induction Motor

A 3-phase wound rotor induction is very similar in construction to a 3-phase transformer. Thus, the motor has 3 identical primary windings and 3 identical secondary windings-one set for each phase. On account of the perfect symmetry, we can consider a single primary winding and single secondary winding in analysing the behaviour of the motor.

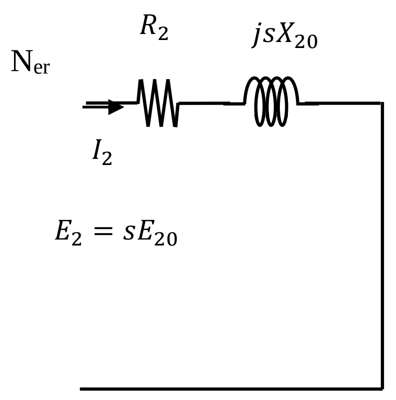
When the motor is at standstill, it acts like a exactly a conventional transformer, and so its equivalent circuit is the same as that of a transformer

Equivalent Circuit of stator



R_1: Resistance of the stator winding /ph
X_1: Leakage reactance of the stator/ph
R_o: Iron loss component
X_m: Magnetizing component
E_1: Induced emf in the stator winding
Nes : Effective turns of stator

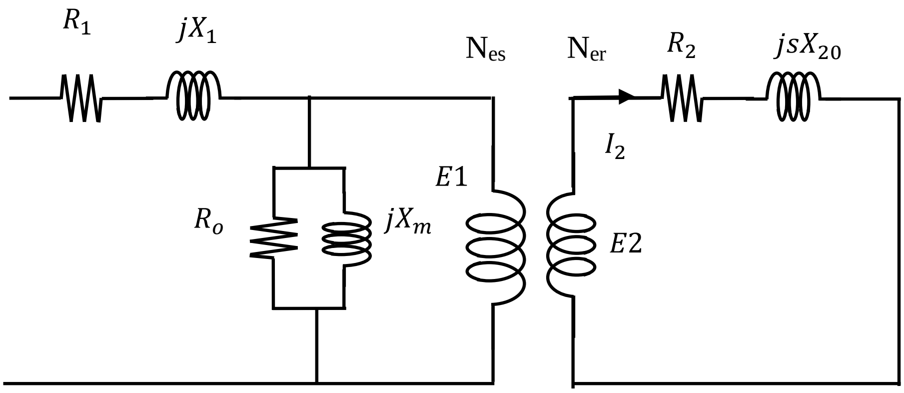
Equivalent Circuit of Rotor



R_2: Rotor resistance /ph
X_{20}: Leakage reactance of the rotor /ph
E_{20}: Emf induced in the rotor /ph
I_2: Rotor current/ph
E_2: Emf inducted in the rotor during running $=sE_{20}$

X_2: Leakage reactance of the rotor during running $=sX_{20}$

❖ **Exact equivalent circuit of the induction motor**

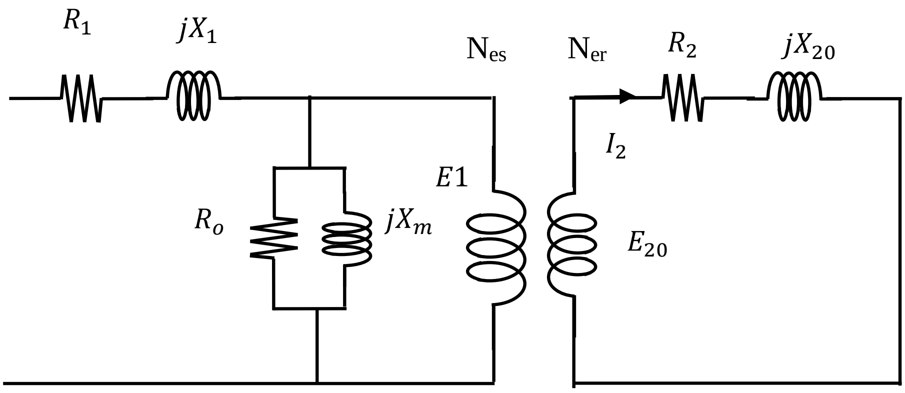


The secondary side loop is excited by a voltage sE_{20}, which is also at a frequency sf_1. This is the reason why the rotor

$$I_2 = \frac{sE_{20}}{\sqrt{{R_2}^2 + (sX_{20})^2}}$$

This expression can be modified as follows (dividing numerator and denominator by s)

$$I_2 = \frac{E_{20}}{\sqrt{\left(\frac{R_2}{s}\right)^2 + (X_{20})^2}}$$



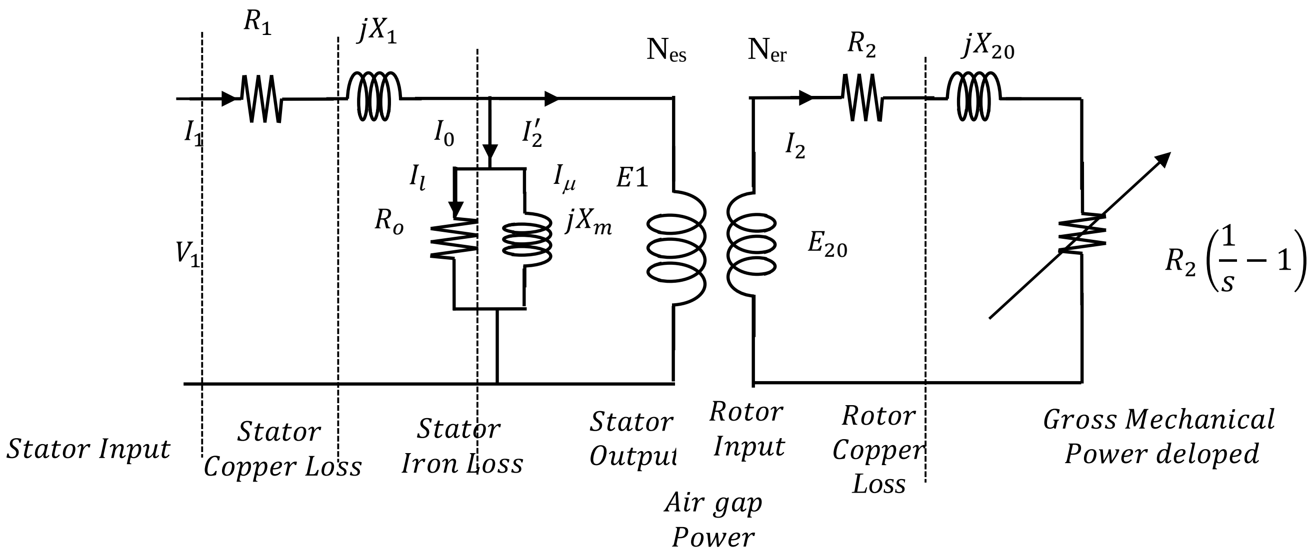
Now the resistance $\frac{R_2}{s}$ can be written as $R_2 + R_2\left(\frac{1}{s} - 1\right)$. It consists of two parts

(i) The first part R2 is the rotor resistance itself and represents the rotor copper loss

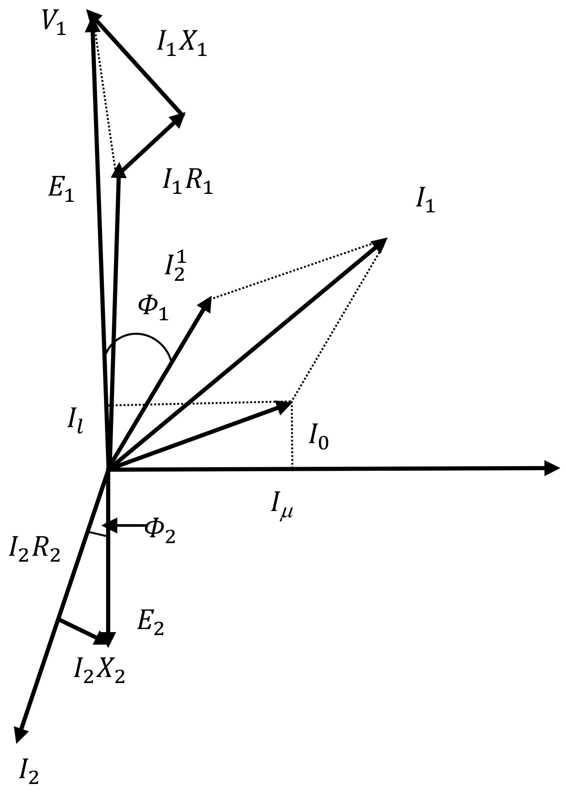
(ii) The second part is $R_2\left(\frac{1}{s} - 1\right)$

$R_2\left(\frac{1}{s} - 1\right)$ is known as the load resistance R_L and is the electrical equivalent of the mechanical load placed on the motor shaft. In other words, the mechanical load on

an induction motor can be represented by a non-inductive resistance of the value $R_2\left(\frac{1}{s}-1\right)$



Phasor Diagram of Induction Motor



❖ Comparisons between transformer and induction motors

I_μ = 4 to 6% of I_{fl}	I_μ = 25 to 35% of I_{ful}
I_l = 1 to 2% of I_{fl}	I_l = 5% of I_{full}
I_0 = 5 to 8% of I_{fl}	I_0 = 30% to 40% of I_{ful}
No- load pf angle ϕ_0 = 70 to 75	I_μ = 5 to 7 I_l
$\cos\phi_0 \equiv 0.2$ lag	No- load pf angle $\phi_0 =\equiv 80$ to 85^0
I_μ = 3 to 4 time I_l	$\cos\phi_0 = 0.1$ lagging

Reactive component of current is more in case of induction motor compared to T/F

$K = \frac{E_2/Ph}{E_1/Ph} = \frac{N_2/Ph}{N_1/Ph}$ in case of 3ϕ T/f

But " K" for ship ring I_M

$E_1/P_h = 4.44\, N_{phs}\, \phi_R\, f\, K_{ws}$

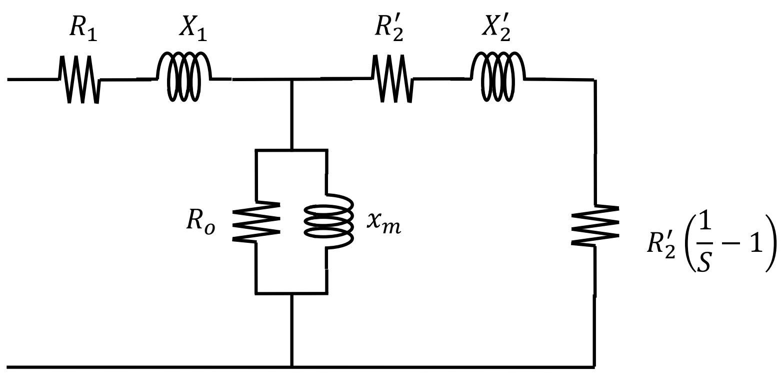
$E_2/Ph = 4.44\, N_{Pht}\, \phi_R\, f\, K_{wr}$

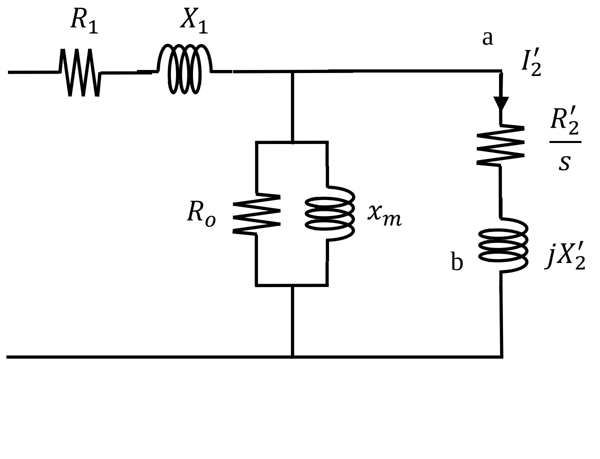
$K = \frac{E_2/Ph}{E_1/ph} = \frac{N_{phr}K_{wr}}{N_{Qns}K_{ws}} = \frac{N_{er}}{N_{es}}$

Effective No. of turns = No of turns/phase x winding factor.

❖ Equivalent circuit of the Induction motor referred to stator side

As in the case of transformer, in this case also, the secondary values may be transferred to the primary and vice versa. As before, it should be remembered that when shifting impedance or resistance from secondary to primary, it should be dived by K^2 whereas current should be multiplied by K. The equivalent circuit of an induction motor all values have been referred to stator side is shown in the following figure





This is then the per-phase equivalent circuit of the induction machine, also called as exact equivalent circuit. Note that the voltage coming across the magnetizing branch is the applied stator voltage, reduced by the stator impedance drop. Generally the stator impedance drop is only a small fraction of the applied voltage.

From the equivalent circuit, one can see that the dissipation in R_1 represents the stator loss, and dissipation in R_0 represents the iron loss. Therefore, the power absorption indicated by the rotor part of the circuit must represent all other means of power consumption - the actual mechanical output, friction and windage loss components and the rotor copper loss components. Since the dissipation in R_2' is rotor copper loss, the power dissipation in R_2'(1 − s)/s is the sum total of the remaining. In standard terminology, dissipation in

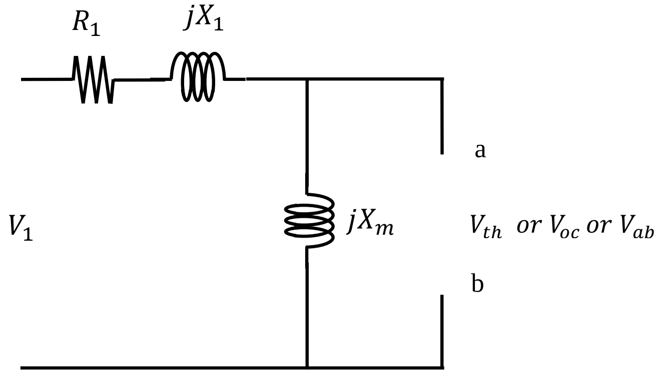
$$\frac{R_2'}{s}\ is\ called\ the\ air\ gap\ power$$

$$R_2'\ is\ calle\ the\ rotor\ copper\ loss$$

$$R_2'\left(\frac{1}{s}-1\right) is\ called\ the\ mechanical\ power$$

- ***Analysis of Equivalent Circuit-Exact Torque Equation***

Step 1: Remove the branch $\frac{R_2'}{s}+jX_2'$ and the assign the terminals as "a" and "b" with voltage across "a" and "b" as V_{th} or V_{oc} or V_{ab} as shown in the figure



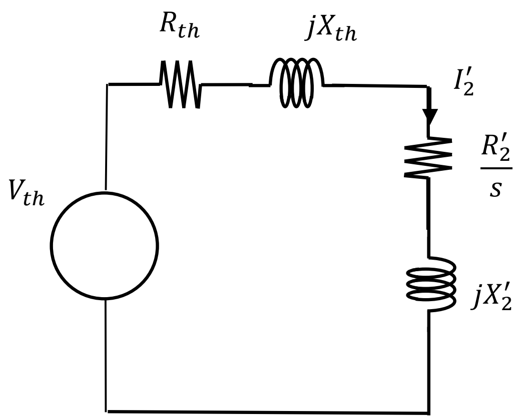
Step 2: Calculate V_{th} and Z_{th}

$$V_{th}=\frac{|V_1|jX_m}{R_1+j(X_1+X_m)}$$ As per Voltage division rule

$$Z_{th}=R_{th}+jX_{th}=\frac{jX_m\times(R_1+jX_1)}{R_1+j(X_1+X_m)}$$

Where R_{th} is the real part of Z_{th} and X_{th} is the imaginary part of Z_{th}

Step 3: Draw the thevenin's equivalent circuit and the connect the branch $\frac{R_2'}{s}+jX_2'$ which is removed in step 1 and calculate the current I_2' using the following equation



$$I_2' = \frac{V_{th}}{\left[\left(R_{th} + \frac{R_2'}{s}\right) + j(X_{th} + X_2')\right]}$$

$$I_2' = \frac{V_{th}}{\sqrt{\left[\left(R_{th}+\frac{R_2'}{s}\right)^2 + (X_{th}+X_2')^2\right]}}$$

Step 4: Using I_2' from step 3 calculate the Exact torque equation

Basic Torque Equation is $T = \frac{180}{2\pi N_s} I_2'^2 \frac{R_2'}{s}$

Substituting the value I_2' in the basic torque equation

$$T = \frac{180}{2\pi N_s}\left(\frac{V_{th}}{\sqrt{\left[\left(R_{th} + \frac{R_2'}{s}\right)^2 + \left(X_{th} + X_2'\right)^2\right]}}\right)^2 \frac{R_2'}{s}$$

$$T = \frac{180}{2\pi N_s}\frac{V_{th}^2}{\left[\left(R_{th}+\frac{R_2'}{s}\right)^2 + \left(X_{th}+X_2'\right)^2\right]}\frac{R_2'}{s}$$ Which is

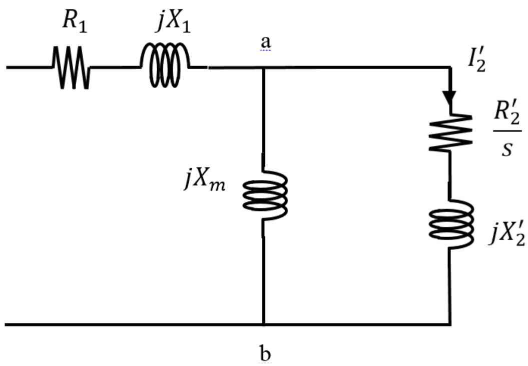
Exact Torque Equation

Model 1: Calculating full load torque or fractional load torque and Efficiency

Step 1: Collect the information given in the problem, and calculate the following

Given data, in general have
R_1(Stator winding resistance), R_2'(Rotor winding resistance referred to stator), X_1(Leakage reactance of stator), X_2'(leakage reactance of rotor referred to stator side), X_m(Magnetizing reactance), Supply voltage V_1,Supply frequency f, Motor poles "p" and rotational losses(Fixed losses)
From this calculate the following
$N_s = \frac{120f}{f}$ and
slip if N_r is given using $s = \frac{N_s - N_r}{N_s}$

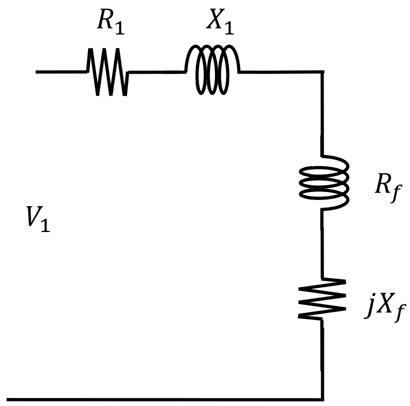
Step 2: Draw the equivalent circuit of the induction motor referred to stator by sub-stuting all the given parameters



Let $Z_1=R_1+jX_1$, $Z_2=jX_m$ and $Z_3=\frac{R_2'}{s}+jX_2'$ and let the impedance across the terminals a-b is Z_f

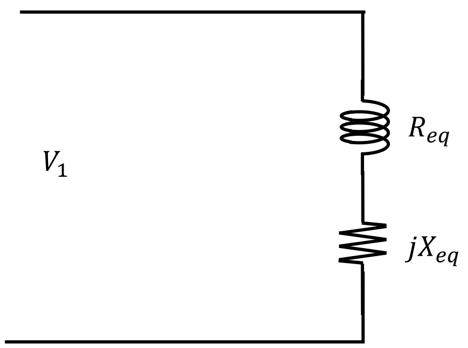
$$Z_f=\frac{jX_m(\frac{R_2'}{s}+jX_2')}{\frac{R_2'}{s}+j(X_2'+X_m)} = R_f + X_f$$

Where R_f is the real part of Z_f and X_f is the imaginary part of Z_f then the equivalent circuit becomes



Zeq= $Z_1+Z_f= R_1+jX_1+R_f+jX_f$

Then the equivalent circuit becomes



Then calculate the stator current (Input current) and Input power factor using the following equations

$$I_1 = \frac{\overline{V_1}}{Z_{eq}} = \frac{V_1\lfloor 0}{z_{eq}\lfloor\theta} = \left(\frac{V_1}{z_{eq}}\lfloor -\theta\right)$$

Therefore the Magnitude of the stator current is $\frac{V_1}{z_{eq}}$ and the input power factor angle is

θ , here "-"indicates that the input current is lagging and the power factor is a lagging power factor

Step 3: Using the above values calculate the following quantities

(i) *Stator Input* $P_{si}=3V_1I_1Cos\theta$

(ii) *Air gap power* $P_{ag}=3I_1^2R_f$

(iii) *Rotor Copper loss*: sP_{ag}

(iv) Torque=$\frac{P_{ag}}{\omega_s}$

(v) Gross mechanical power developed $P_{gmd}=P_{ag}(1- s)$

(vi) Net mechanical power $P_{net}= P_{gmd}$-*Fixed losses*

(vii) $\eta = \frac{P_{net}}{P_{si}} \times 100$

(viii) $T_{sh} = \frac{P_{net}}{\omega_r}$

A 3ϕ, star connected, 400V, 50Hz, 4 pole IM has the following phase constants in ohms referred to stator:

R_1 = 0.15, X_1 =X_2'= 0.45, R_2' = 0.12, X_m = 28.5

Fixed losses (core, friction & winding losses) = 400 watt. Compute the stator current, rotor speed, output torque & efficiency when motor is operated at rated voltage and frequency at a slip of 4%

A 3ϕ, star connected, 400V, 50Hz, 4 pole IM has the following phase constants in ohms referred to stator on equivalent star basis:

$R_1 = 1.2$, $X_1 = 1.16$, $R_2' = 0.4$, $X_2' = 1.16$ $X_m = 28.5$

Fixed losses (core, friction & winding losses) = 800 watt. Compute the stator current, slip, output torque & efficiency when motor is operated at rated voltage and frequency for a speed of 1440rpm (Ans: 21.1 A,0.04,78.98Nm,90.5%)

Using Thevenin's Theorem

Model 1: Calculating full load torque or fractional full load torque

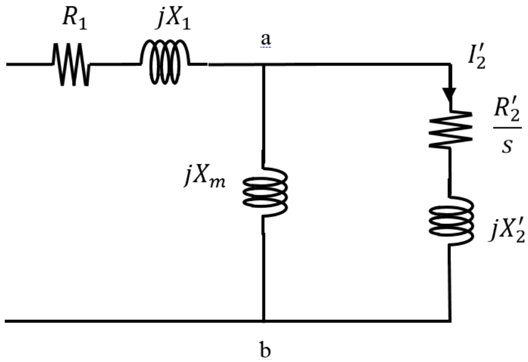
$$\mathrm{T} = \frac{180}{2\pi \mathrm{N_s}} \mathrm{I}'^2_{2\mathrm{fl}} \frac{\mathrm{R}'_2}{s_{fl}}$$

$$I'_{2fl} = \frac{V_{th}}{\sqrt{\left[\left(R_{th} + \frac{R'_2}{s_{fl}}\right)^2 + (X_{th} + X'_2)^2\right]}}$$

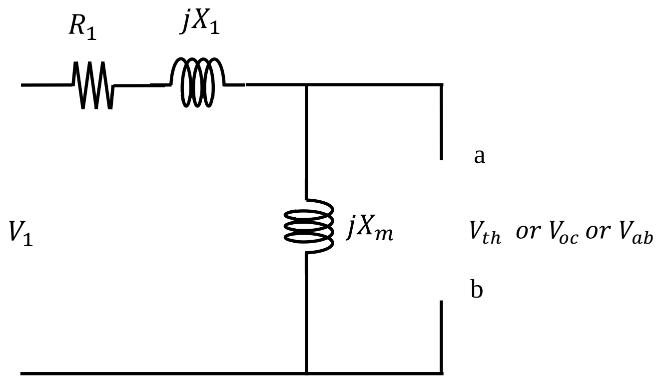
Step 1: Collect the information given in the problem, and calculate the following

Given data, in general have
R_1(Stator winding resistance), R_2'(Rotor winding resistance referred to stator), X_1(Leakage reactance of stator), X_2'(leakage reactance of rotor referred to stator side), X_m(Magnetizing reactance), Supply voltage V_1,Supply frequency f, Motor poles "p" and rotational losses(Fixed losses)
From this calculate the following
$N_s = \frac{120f}{f}$ and
slip if N_r is given using $\mathrm{s} = \frac{\mathrm{N_s} - \mathrm{N_r}}{\mathrm{N_s}}$

Step 2: Draw the equivalent circuit of the induction motor referred to stator by substuting all the given parameters as shown in the figure



Remove the branch $\frac{R'_2}{s} + jX'_2$ and the assign the terminals as "a" and "b" with voltage across "a" and "b" as V_{th} or V_{oc} or V_{ab} as shown in the figure



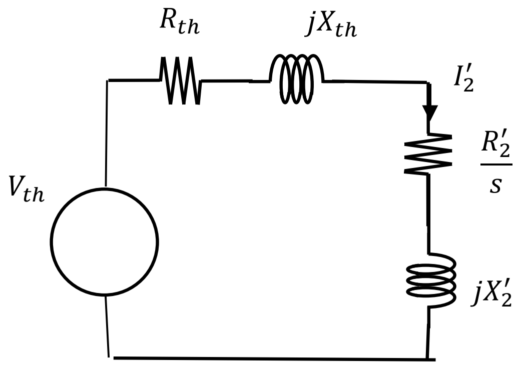
Step 3: Calculate V_{th} and Z_{th}

$$V_{th} = \frac{|V_1| jX_m}{R_1 + j(X_1 + X_m)} \qquad \text{As per Voltage division rule}$$

$$Z_{th} = R_{th} + jX_{th} = \frac{jX_m \times (R_1 + jX_1)}{R_1 + j(X_1 + X_m)}$$

Where R_{th} *is the real part of* Z_{th} *and* X_{th} *is the imaginary part of* Z_{th}

Step 4: Draw the thevenin's equivalent circuit and the connect the branch $\frac{R_2'}{s} + jX_2'$ which is removed in step 1 and calculate the current I_2' using the following equation



$$I_{2fl}' = \frac{V_{th}}{\left[\left(R_{th} + \frac{R_2'}{s_{fl}}\right) + j(X_{th} + X_2')\right]}$$

$$I_{2fl}' = \frac{V_{th}}{\sqrt{\left[\left(R_{th} + \frac{R_2'}{s_{fl}}\right)^2 + (X_{th} + X_2')^2\right]}}$$

Step 5: using the above values calculate the following

(i) $\mathrm{T} = \frac{180}{2\pi \mathrm{N_s}} \mathrm{I}_{2\mathrm{fl}}'^2 \frac{\mathrm{R}_2'}{s_{fl}}$

(ii) $P_{ag} = 3\mathrm{I}_{2\mathrm{fl}}'^2 \frac{\mathrm{R}_2'}{s_{fl}}$

(iii) $P_{gmd} = P_{ag}(1 - s_{fl})$

(iv) $P_{net} = P_{gmd} - Fixed\ Losses$

A 3ϕ, star connected, 400V, 50Hz, 4 pole IM has the following phase constants in ohms referred to stator:

$R_1 = 0.15$, $X_1 = X_2' = 0.45$, $R_2' = 0.12$, $X_m = 28.5$

Compute full load torque when motor is operated at rated voltage and frequency for a rated speed of 1440rpm

Model 2: Calculating Maximum torque, Slip at Maximum torque and corresponding speed.

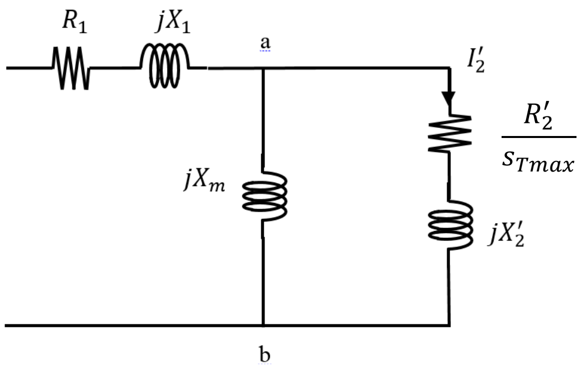
$$T_{max} = \frac{180}{2\pi N_s} I'^2_{2Tmax} \frac{R'_2}{s_{Tmax}}$$

$$I'_{2Tmax} = \frac{V_{th}}{\sqrt{\left[\left(R_{th} + \frac{R'_2}{s_{Tmax}}\right)^2 + (X_{th} + X'_2)^2\right]}}$$

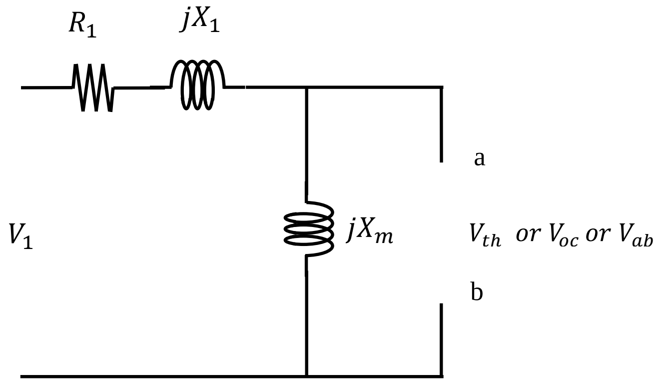
Step 1: Collect the information given in the problem, and calculate the following

Given data, in general have
R_1(Stator winding resistance), R_2'(Rotor winding resistance referred to stator), X_1(Leakage reactance of stator), X_2'(leakage reactance of rotor referred to stator side), X_m(Magnetizing reactance), Supply voltage V_1,Supply frequency f, Motor poles "p" and rotational losses(Fixed losses)
From this calculate
$N_s = \frac{120f}{f}$ and the information about the slip at maximum torque is not provided, which has to be calculated for maximum torque

Step 2: Draw the equivalent circuit of the induction motor referred to stator by sub-stuting all the given parameters as shown in the figure



Remove the branch $\frac{R_2'}{s_{Tmax}} + jX_2'$ and the assign the terminals as "a" and "b" with voltage across "a" and "b" as V_{th} or V_{oc} or V_{ab} as shown in the figure



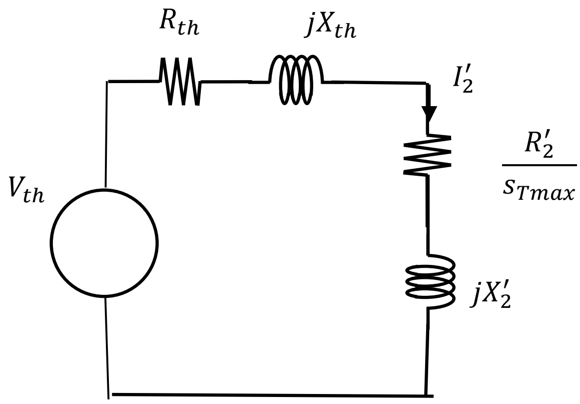
Step 3: Calculate V_{th} and Z_{th}

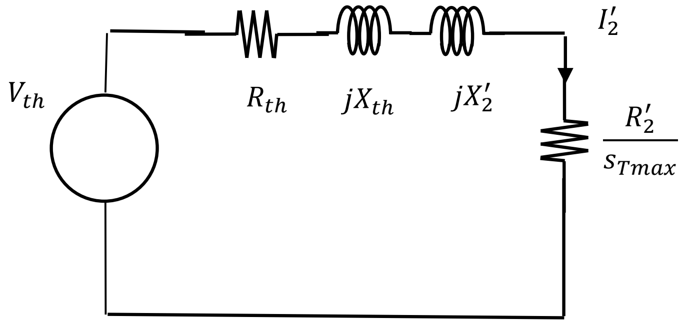
$$V_{th} = \frac{|V_1| jX_m}{R_1 + j(X_1 + X_m)}$$ As per Voltage division rule

$$Z_{th} = R_{th} + jX_{th} = \frac{jX_m \times (R_1 + jX_1)}{R_1 + j(X_1 + X_m)}$$

Where R_{th} *is the real part of* Z_{th} *and* X_{th} *is the imaginary part of* Z_{th}

Step 4: Draw the thevenin's equivalent circuit and connect the branch $\frac{R_2'}{s_{Tmax}} + jX_2'$ which is removed in step 1 and calculate the slip corresponding to maximum torque using maximum power transfer theorem





$$\frac{R_2'}{s_{Tmax}} = \sqrt{R_{th}^2 + (X_{th} + X_2^1)}^{2}$$

$$S_{Tmax=} \frac{R_2^{'}}{\sqrt{R_{th}^2 + (X_{th} + X_2^1)^2}}$$

$$N_{rTmax} = N_s(1 - S_{Tmax})$$

Step 5 : Using the calculated values of V_{th}, R_{th}, X_{th} and S_{Tmax} calculate the current at Maximum torque and Maximum Torque using the following equations

$$I_{2Tmax}' = \frac{V_{th}}{\sqrt{\left[\left(R_{th} + \frac{R_2'}{s_{Tmax}}\right)^2 + (X_{th} + X_2')^2\right]}}$$

$$T_{max} = \frac{180}{2\pi N_s} I_{2Tmax}'^2 \frac{R_2'}{s_{Tmax}}$$

For the above problem calculate the slip at maximum torque, speed and maximum torque

Model 3: Calculating starting torque

$$T_{st} = \frac{180}{2\pi N_s} I'^2_{2st} \frac{R'_2}{(1)}$$

$$I'_{2st} = \frac{V_{th}}{\sqrt{\left[\left(R_{th} + \frac{R'_2}{(1)}\right)^2 + (X_{th} + X'_2)^2\right]}}$$

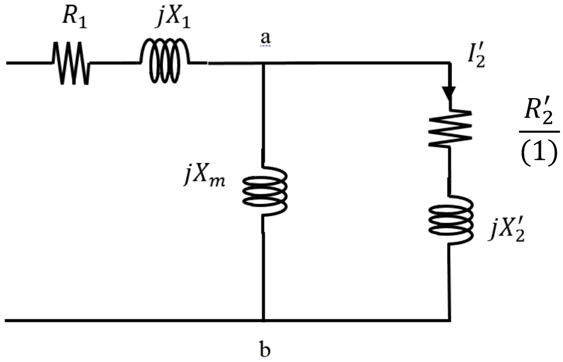
Step 1: Collect the information given in the problem, and calculate the following

Given data, in general have
R_1(Stator winding resistance), R_2'(Rotor winding resistance referred to stator), X_1(Leakage reactance of stator), X_2'(leakage reactance of rotor referred to stator side), X_m(Magnetizing reactance), Supply voltage V_1,Supply frequency f, Motor poles "p" and rotational losses(Fixed losses)

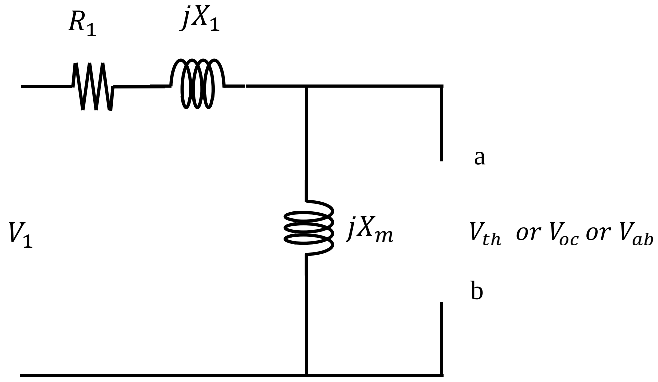
From this calculate

$N_s = \frac{120f}{f}$ and the value of slip has to be taken as “1”

Step 2: Draw the equivalent circuit of the induction motor referred to stator by sub-stuting all the given parameters as shown in the figure



Remove the branch $\frac{R_2'}{s_{Tmax}} + jX_2'$ and the assign the terminals as “a” and “b” with voltage across “a” and “b” as V_{th} or V_{oc} or V_{ab} as shown in the figure



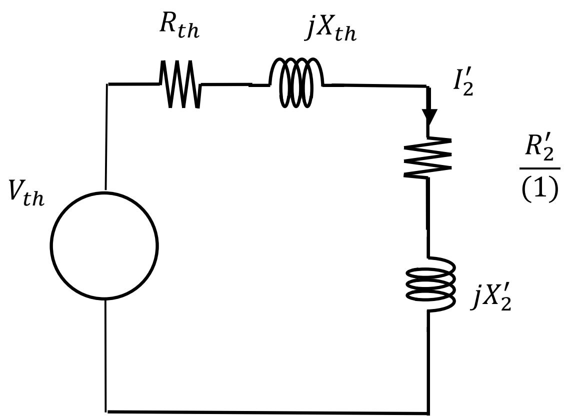
Step 3: Calculate V_{th} and Z_{th}

$$V_{th} = \frac{|V_1| jX_m}{R_1 + j(X_1 + X_m)}$$ As per Voltage division rule

$$Z_{th} = R_{th} + jX_{th} = \frac{jX_m \times (R_1 + jX_1)}{R_1 + j(X_1 + X_m)}$$

Where R_{th} *is the real part of* Z_{th} *and* X_{th} *is the imaginary part of* Z_{th}

Step 4: Draw the thevenin’s equivalent circuit and connect the branch $\frac{R_2'}{(1)} + jX_2'$ which is removed in step 1 and calculate starting current and torque using the following equations



$$I'_{2st} = \frac{V_{th}}{\sqrt{\left[\left(R_{th} + \frac{R'_2}{(1)}\right)^2 + (X_{th} + X'_2)^2\right]}}$$

$$T_{st} = \frac{180}{2\pi N_s} I'^2_{2st} \frac{R'_2}{(1)}$$

For the above problem calculate the starting current and starting torque

Model 4: Calculating the additional resistance required in the rotor circuit to obtain maximum torque at starting

Let R'_{ext} be the additional resistance referred to stator be added in the rotor circuit to obtain maximum torque at starting

From model 2 , step 4 we have the maximum slip corresponding to maximum given by the following equation

$$S_{Tmax=} \frac{R_2^{'}}{\sqrt{R_{th}^2 + (X_{th} + X_2^1)^2}}$$

R'_{ext} be added in the rotor circuit to obtain the maximum torque at starting

Then

$$S_{Tmax=}\ 1 = \frac{R_2^{'} +\ R_{ext}^{'}}{\sqrt{R_{th}^2 + (X_{th} + X_2^1)^2}}$$

$$\text{Then } \boldsymbol{R'_{ext}} = \sqrt{R_{th}^2 + (X_{th} + X_2^1)^2} \ - \ R_2'$$

Then the value of external resistance referred to rotor R_{ext}=K^2 $\boldsymbol{R'_{ext}}$

For the above problem calculate external resistance to be inserted in the rotor circuit to produce maximum torque at starting, Stator to rotor effective turns ratio is 1.2

Model 5: Calculating Maximum mechanical power, Slip at Maximum mechanical and corresponding speed.

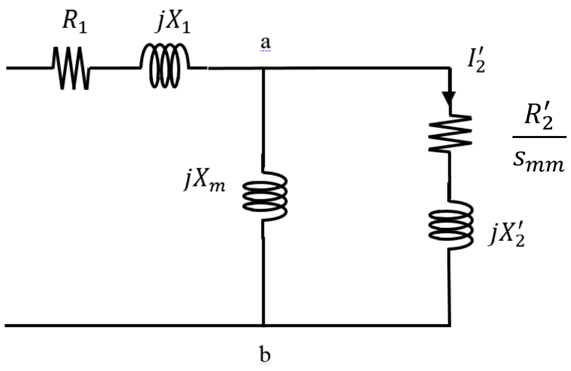
$$P_{gmdmax} = 3I_{2mm}^2 R_2^1 \left(\frac{1}{s} - 1\right)$$

$$I'_{2mm} = \frac{V_{th}}{\sqrt{\left[\left(R_{th} + \frac{R'_2}{s_{mm}}\right)^2 + \ (X_{th} + \ X'_2)^2\right]}}$$

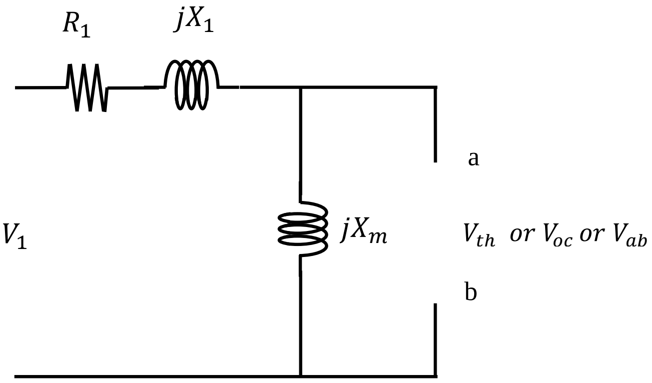
Step 1: Collect the information given in the problem, and calculate the following

Given data, in general have
R_1(Stator winding resistance), R_2'(Rotor winding resistance referred to stator), X_1(Leakage reactance of stator), X_2'(leakage reactance of rotor referred to stator side), X_m(Magnetizing reactance), Supply voltage V_1,Supply frequency f, Motor poles "p" and rotational losses(Fixed losses)
From this calculate
$N_s = \frac{120f}{f}$ and the information about the slip at maximum mechanical power is not provided, which has to be calculated for maximum mechanical power

Step 2: Draw the equivalent circuit of the induction motor referred to stator by sub-stuting all the given parameters as shown in the figure



Remove the branch $\frac{R_2'}{s_{Tmax}} + jX_2'$ and the assign the terminals as "a" and "b" with voltage across "a" and "b" as V_{th} or V_{oc} or V_{ab} as shown in the figure



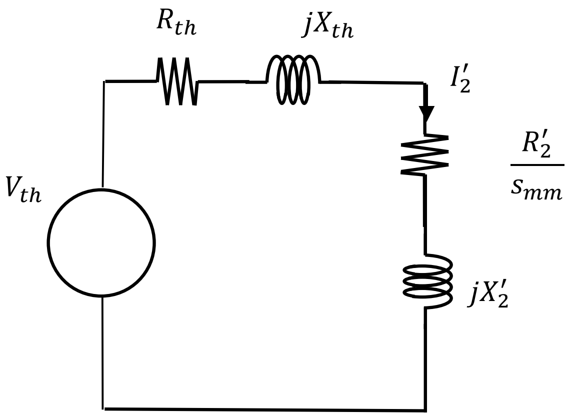
Step 3: Calculate V_{th} and Z_{th}

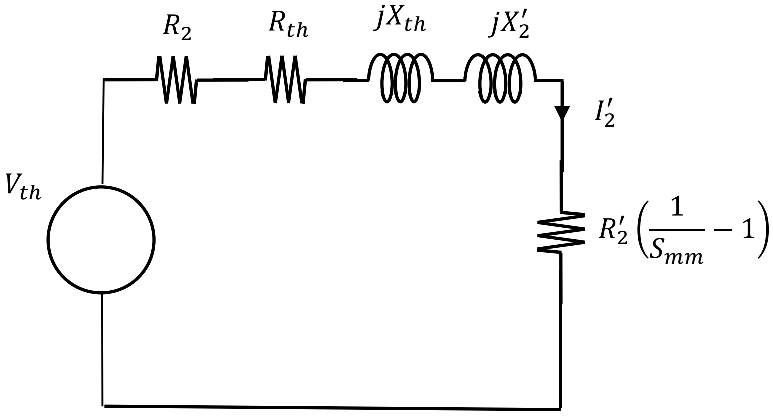
$$V_{th} = \frac{|V_1| jX_m}{R_1 + j(X_1 + X_m)} \qquad \text{As per Voltage division rule}$$

$$Z_{th} = R_{th} + jX_{th} = \frac{jX_m \times (R_1 + jX_1)}{R_1 + j(X_1 + X_m)}$$

Where R_{th} *is the real part of* Z_{th} *and* X_{th} *is the imaginary part of* Z_{th}

Step 4: Draw the thevenin's equivalent circuit and connect the branch $\frac{R_2'}{S_{mm}} + jX_2'$ which is removed in step 1 and calculate the slip corresponding to maximum torque using maximum power transfer theorem





$$R_2'\left(\frac{1}{S_{mm}} - 1\right) = \sqrt{(R_2' + R_{th})^2 + (X_{th} + X_2^1)^2}$$

$$S_{mm} = \frac{R_2'}{R_2' + \sqrt{(R_2' + R_{th})^2 + (X_{th} + X_2^1)^2}}$$

$$N_{rmm} = N_s(1 - S_{mm})$$

Step 5 : Using the calculated values of V_{th}, R_{th}, X_{th} and S_{mm} calculate the current at Maximum mechanical power and Maximum mechanical power using the following equations

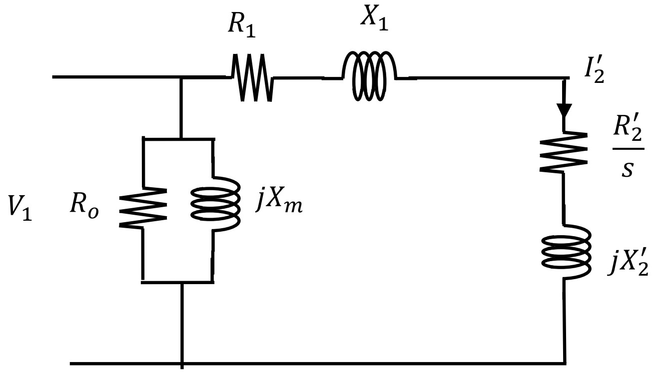
$$I_{2mm}' = \frac{V_{th}}{\sqrt{\left[\left(R_{th} + \frac{R_2'}{s_{mm}}\right)^2 + (X_{th} + X_2')^2\right]}}$$

$$P_{gmdmax} = 3I_{2mm}^2 R_2^1\left(\frac{1}{s} - 1\right)$$

For the above problem calculate the maximum internal power developed and the corresponding slip

Approximate Equivalent Circuit

In the case of a conventional 3-phase transformer, we would be justified in removing the magnetizing branch composed of jX_m and R_o because the exciting current I_0 is negligible compared to the load current. However in a motor this is no longer true: I_0 may be as high as 40% of load current because of air gap. Consequently we cannot eliminate the magnetizing branch. However, for motor exceeding 2hp, we can shift it to the input terminals, as shown in the figure. This greatly simplifies the equations that describe the behaviour of the motor, with out compromising accuracy



In all the above five model, if the approximate equivalent circuit model is assumed there is no need to find out thevenin's voltage, thevenin's resistance and thevenin's reactance.

In place use, supply voltage/ph, stator winding resistance value and stator reactance value

Losses and efficiency

Every electro-mechanical energy conversion is accompanied by losses. Knowing losses is required because of two reasons. Firstly, the losses influence the operating cost of the machine. Secondly the losses determine the heating of the machine and hence fix the machine rating or power output, which can be obtained without deterioration of the insulation due to over-heating. The various losses in rotating machines are:

(a) ***Copper-losses or I^2R loss in the rotor and stator windings***. To calculate the losses the resistance of the windings should be taken at the operating temperature, which is assumed to be 75° C.

(b) ***Iron losses, i.e hysteresis and eddy current losses***. These losses are constant.

(c) ***Mechanical or frictional and windage losses***. These losses are also constant unless speed varies appreciably.

(d) ***Stray load losses***, which mean additional hysteresis and eddy current losses arising from any distortion in flux distribution caused by the load current. These losses are difficult to measure and are usually taken as 1% of the machine output.

$$\text{Efficiency} = \frac{\text{Power output}}{\text{Power input}}$$

For a motor, especially the large ones, it is difficult to measure mechanical power output. Therefore

$$\text{Efficiency of motor} = \frac{\text{Power input} - \text{losses}}{\text{Power input}}$$

For generator, it is almost impossible to measure power input. Therefore

$$\text{Efficieny of generator} = \frac{\text{Power output}}{\text{Power output} + \text{losses}}$$

(a) Copper losses

Total copper losses of induction machine in per phase with respect to stator = $3I_1^2R_{01}$, where I_1 is full load stator current in per phase and R_{01} is total resistance in per phase with respect to stator side and $R_{01}/ph = R_1\Big/ph + \dfrac{R_2/ph}{K^2} = R_1 + R_2'$

Similarly total copper losses of induction machine in per phase with respect to rotor = $3I_2^2R_{02}$, where I_2 is full load rotor current in per phase and R_{02} is total resistance in per phase with respect to rotor side and $R_{02}/ph = R_2/ph + R_1 K^2 = R_2 + R_1'$

Similarly total copper losses of induction machine in per phase $=3(I_1^2R_1 + I_2^2R_2)$, where I_1 is the full load stator current and I_2 is the full load rotor current, and R_1 and R_2 are the resistances of stator and rotor winding per phase respectively.

The copper losses at full load current are called full load copper losses and copper losses are $\alpha\ I_1^2\Big/ph$, therefore the copper losses are also called variable losses.

Copper losses at x of full load = x^2 × full load Copper losses

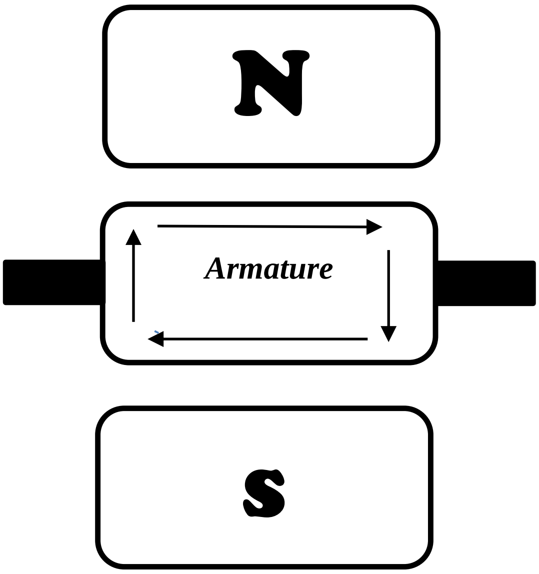
e.g If the full load copper losses are 1200 watts, then the copper losses at half full load will be $\left(\frac{1}{2}\right)^2 \times 1200 = 300\ watts$

(b) Iron losses

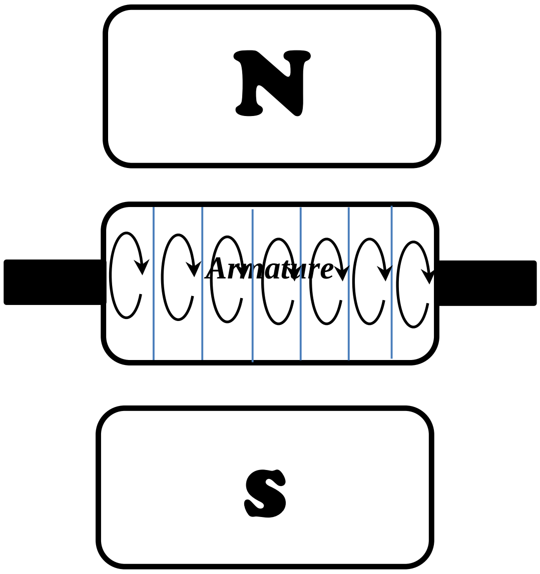
(i) Eddy Current loss

The core of a generator armature is made from iron, which is a conducting material with desirable magnetic characteristics. Any conductor will have currents induced in it when it is rotated in a magnetic field. These currents that are induced in the generator armature core are called ***EDDYCURRENTS***. The power dissipated in the form of heat, as a result of the eddy currents, is considered a loss and this loss can be called as eddy current loss

Eddy currents, just like any other electrical currents, are affected by the resistance of the material in which the currents flow. The resistance of any material is inversely proportional to its cross-sectional area. Figure shows, view (a), shows the eddy currents induced in an armature core that is a solid piece of soft iron. Figure view (b), shows a soft iron core of the same size, but made up of several small pieces insulated from each other. This process is called lamination. The currents in each piece of the laminated core are considerably less than in the solid core because the resistance of the pieces is much higher. (Resistance is inversely proportional to cross-sectional area.) The currents in the individual pieces of the laminated core are so small that the sum of the individual currents is much less than the total of eddy currents in the solid iron core



(a)

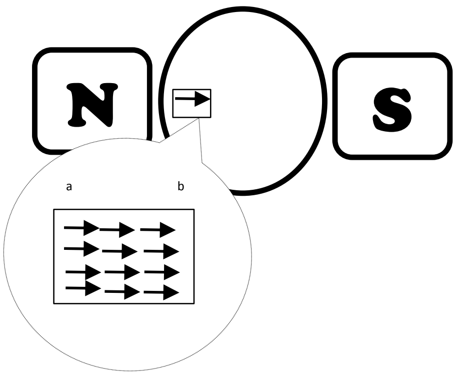


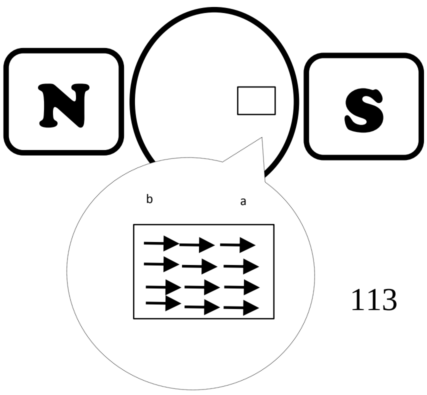
(b)

Eddy current loss:$\boldsymbol{P_e \propto B^2 f^2}$

(ii) Hysteresis Losses

Steel is very good ferromagnetic material. This kind of materials are very sensitive to be magnetized. That means whenever magnetic flux passes through, it will behave like magnet. Ferromagnetic substances have numbers of domains in their structure. Domain are very small region in the material structure, where all the dipoles are paralleled to same direction. In other words, the domains are like small permanent magnet situated randomly in the structure of substance. These domains are arranged inside the material structure in such a random manner in the absent of magnet field, that net resultant magnetic field of the said material is zero. Whenever external magnetic field or mmf is applied to that substance, these randomly directed domains (small magnet/molecular magnets) are arranged themselves in parallel to the axis of applied mmf. After removing this external mmf, maximum numbers of domains again come to random positions, but some few of them still remain in their changed position. Because of these unchanged domains the substance becomes slightly magnetized permanently. This magnetism is called “Spontaneous Magnetism". To neutralize this magnetism some opposite mmf is required to be applied. The magneto motive force or mmf applied of induction machine stator core and rotor core is alternating. For every cycle, due to this domain reversal there will be extra work done. For this reason, there will be a consumption of electrical energy which is known as Hysteresis loss of induction machine





Hysteresis loss: $P_h \propto B^2 f^{1.6}$

Silicon steel, often called electrical steel, is steel with silicon added to it. The addition of silicon to steel increases its electrical resistance, improves the ability of magnetic fields to penetrate it, and reduces the steel's hysteresis loss

(i) Stator core losses

(ii) Rotor core losses

- Stator core losses depend on supply frequency, whereas rotor core losses depends on slip frequency.
- Hysteresis losses and Eddy current losses are functions of frequency in Induction machine.
- The rotor frequency (i.e slip frequency) is very less, therefore rotor core losses are very less and ***hence the total Iron losses are stator core losses.***
- As the core losses depends on input voltage and frequency , core losses are electrical losses.

Effect of variable voltage and variable frequency on iron, hysteresis and eddy current losses

Case1: Variable voltage and variable frequency with v/f constant

Case 2: Fixed voltage variable frequency with v/f not constant

Case 3: Variable voltage with fixed frequency

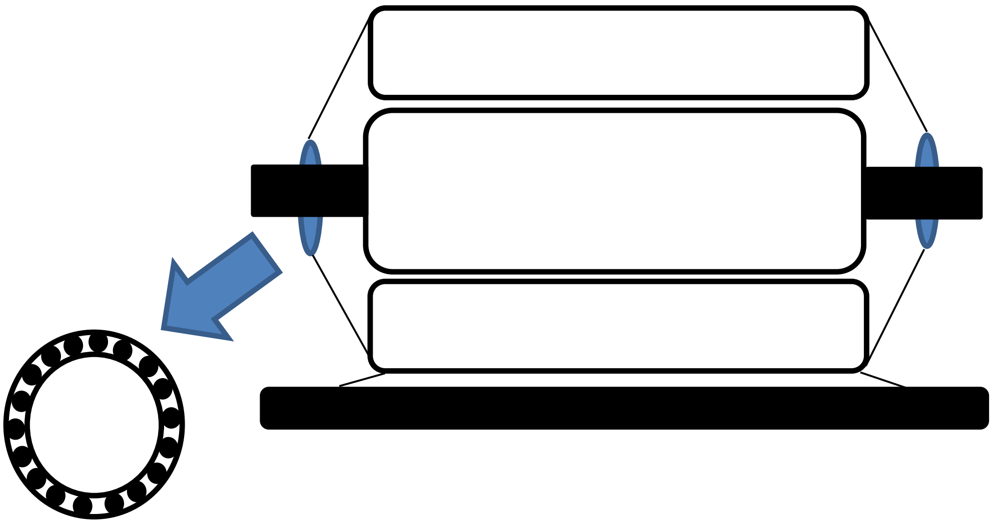
(b) Mechanical losses:

(i) Friction loss
(ii) Windage loss

(a) FRICTION LOSS:

(1) Bearing friction loss (present in both squirrel cage & slip ring motor)
(2) Brush friction loss (Present in only slip ring Induction motor)

Bearing friction loss:
Bearing are fitted both sides of the shaft. Bearing are used to reduce temperature



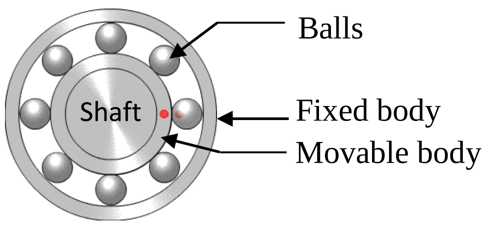


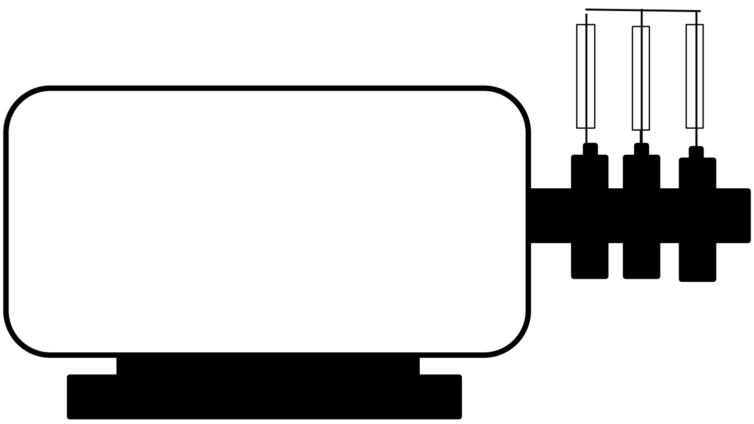
Fig: Ball Bearing

Fig: Roll Bearing

- ❖ Certain lubricant is required. Oil film provides continuous lubrication at the time ball will not touched fixed part. Whenever lubrication is not available balls touch the fixed part. Friction loss takes place between movable or fixed part
- ❖ This loss is proportional to Speed.
- ❖ The friction will occur in between balls and fixed body.
- ❖ Grease is used in bearings to reduce friction in small motors and lubricant oil is used for large motors.

Brush friction loss

Brushes are pressed against slip rings by using slip rings. Slip ring is a rotating part, brush is stationary part. To reduce brush friction loss carbon brushes are used instead of copper.



- ❖ This loss is proportional to $Speed^2$
- ❖ Carbon material is used for brushes as carbon is self-lubricant.
- ❖ Brushes are placed on slip rings.

(b) WINDAGE LOSS

- ❖ Wind offers some resistance to rotation.
- ❖ Windage loss is the mechanical power wasted to overcome opposition offered by wind present in the air gap.
- ❖ This loss is proportional to $Speed^3$.
- ❖ Windage loss will increase with increasing air gap, therefore air gap should be minimum in order to get good performance.
- ❖ If air gap length is more:

- $I_\mu \uparrow$, $\cos\phi_0 \downarrow$, $\cos\phi_{fl} \downarrow$, $X_2 \uparrow$.
- Therefore $T_{max}\downarrow$, $T_{st} \downarrow$, and windage loss is increases.

❖ The air gap length is more in slip ring Induction motor as compared to squirrel cage.

❖ Mechanical losses are more in slip ring Induction motor when compared with squirrel cage induction motor due to following reasons.

- Due to presence of more air gap length.
- Due additional brush friction losses.
  - If speed is increased by 10%,
    Then bearing friction loss increased by 10%
    Brush friction loss (α N^2) increased by 21%
    Windage friction loss (α N^3) increased by 33.3 %

❖ Speed variation in Induction motor is very less from No Load to full load, therefore mechanical losses are almost constant since mechanical losses are proportional to speed.

❖ In dc series motor mechanical losses are variable losses.
Using fly wheel dc series motor started with no load, at no load fly wheel consumes the kinetic energy at the time of starting. In order to save electrical power fly wheel is used in variable speed motors (such as dc series motor).
In dc series motor iron loss is not constant.
$W_n \propto N$.
$W_e \propto N^2$.

❖ As induction motor is almost constant speed power, the mechanical loss which depends on speed and also maintained constant from no load to full load. Hence mechanical losses can be treated as constant loss in induction motor.

Testing of Induction motor

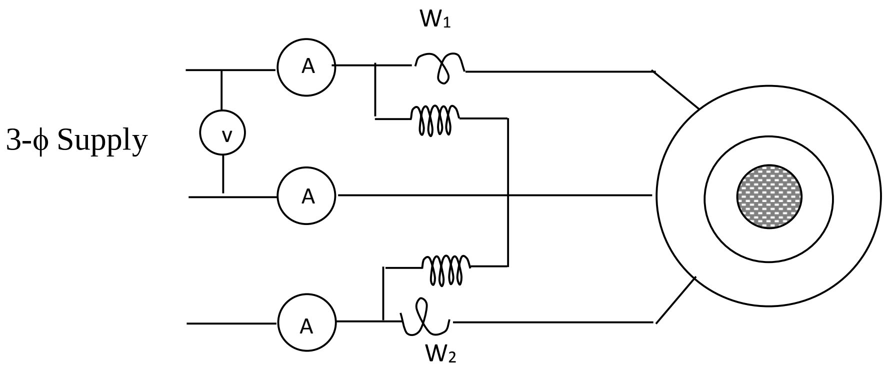
1. No-load test (or) Open circuit test
2. Blocked rotor test (or) Short circuit test (or) Locked rotor test

No load test

Objective:

- To find out shunt branch elements of the equivalent circuit Ro and X_0.
- To find out constant losses of the Induction machine.
- To separate constant losses into Iron losses and mechanical losses.
  - The no load test should be conducted at air gap flux by applying rated voltage and rated frequency to stator under no-load condition of machine.
  - The three phase induction motor under no-load condition just equivalent to a transformer with open circuited secondary that why no-load test is equal to open circuit of transformer.
  - In this test, the power supply system is balanced system then one wattmeter is sufficient otherwise two wattmeter's are required.

Circuit Connection

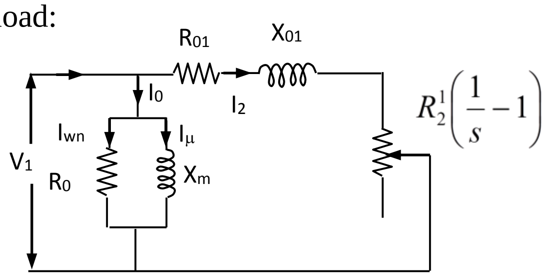


Procedure:

Observation Table

Equivalent Circuit of an Induction motor at no load

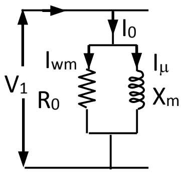
- At No load:



In No-load, slip = 0 and

therefore $R_2^1\left(\frac{1}{S}-1\right)$ is infinite i.e open circuited and I_2 becomes zero.

Now the equivalent circuit is



- Wattmeter reading based on power factor:

Power factor (lagging)	Relation between watt meters
Unity	$w_1 = w_2$
0.866	$w_1 = 2w_2$
0.5	$w_2 = 0$, w_1 = total 3-ϕ power
Zero	$w_1 = -w_2$

When wattmeter shows negative reading, reverse either pressure coil or current coil terminal for getting positive reading.

At No load, power factor is very poor

Calculations:

Let us consider supply system is unbalanced system

V = Line to line voltage.

I_0 = Average of three ammeters line currents.

$W_{oc} = W_1 + W_2$; Three phase power.

To find out R_o and X_o:

Δ- Connected Stator

$V_1/ph = V_1$, $I_0/ph = \dfrac{I_0}{\sqrt{3}}$

$R_{0m}/ph = \dfrac{V_1/ph}{I_{wm}/ph}$; $X_0/ph = \dfrac{V_1/ph}{I_\mu/ph}$

$I\ /ph = (I_0/ph)Cos\phi_0 = \dfrac{I_0}{\sqrt{3}}Cos\phi_0;$

$I_\mu/ph = (I_0/ph)\,Sin\phi_0 = \dfrac{I_0}{\sqrt{3}}Sin\phi_0;$

$R_o/ph = \dfrac{V_1}{\dfrac{I_0}{\sqrt{3}}Cos\phi_0} = \dfrac{\sqrt{3}V_1}{I_0\,Cos\phi_0};$

$X_m/ph = \dfrac{V_1}{\dfrac{I_0}{\sqrt{3}}Sin\phi_0} = \dfrac{\sqrt{3}V_1}{I_0\,Sin\phi_0};$

$W_{oc} = \sqrt{3}V_1I_0Cos\phi_0$

$\Rightarrow Cos\phi_0 = \dfrac{W_0}{\sqrt{3}V_1I_0}$ and

$Sin\phi_0 = \sqrt{1-\cos^2\phi_0}$

Y- Connected Stator

$I_0/ph = I_0$, $V_1/ph = \dfrac{V_1}{\sqrt{3}}$

$R_o/ph = \dfrac{V_1/ph}{I_{wm}/ph}$; $X_m/ph = \dfrac{V_1/ph}{I_\mu/ph}$

$I_l/ph = (I_0/ph)Cos\phi_0 = I_0Cos\phi_0;$

$I_\mu/ph = (I_0/ph)\,Sin\phi_0 = I_0Sin\phi_0;$

$R_o/ph = \dfrac{V_1/\sqrt{3}}{I_0Cos\phi_0} = \dfrac{V_1}{\sqrt{3}I_0\,Cos\phi_0};$

$X_m/ph = \dfrac{V_1/\sqrt{3}}{I_0Sin\phi_0} = \dfrac{V_1}{\sqrt{3}I_0\,Sin\phi_0}$

$W_0 = \sqrt{3}V_1I_0Cos\phi_0$

$\Rightarrow Cos\phi_0 = \dfrac{W_0}{\sqrt{3}V_1I_0}$ and

$Sin\phi_0 = \sqrt{1-\cos^2\phi_0}$

Note: Meter readings are line values and convert them into phase values.

To find out constant loses:

Two watt meters:

Load power factor < 0.5, w_1 = +ve, w_2 = -ve

No-load power factor is very low so in this way $\emptyset_0$ is more. One wattmeter shows negative reading.

W_1=800W, W_2=200W (After reversing current coil/pressure coil terminals)

So W_1-W_2=800-200=600 W.

To find out R_o & X_m parameters:-

$R_o/ph = \frac{V_1/ph}{I_l/ph}$ $\qquad I_l/ph = I_o/ph\ cos\emptyset_0$

$X_m/ph = \frac{V_1/ph}{I_\mu/ph}$ $\qquad I_\mu/ph = I_o/ph\ sin\emptyset_0$

To find out constant losses:-

$W_{0C} \cong$ losses in induction motor under no load condition

Iron losses (stator core losses only); Rotor core losses are negligibly small

W_{oc} = Losses in induction motor under No load condition (mechanical power output = 0)

= Iron losses (stator core losses) + Mechanical losses + No load stator copper losses.

Constant losses = W_{oc} – No load Stator copper losses.

$$= W_{oc} - \left(3I_0^2 R_1 \ \ per\ phase\right)$$

To measure stator resistance($\boldsymbol{R_1/ph}$):

Stator winding resistance(R_1/ph) can be measured by using kelvin double bridge method.

To separate constant losses into Iron and Mechanical losses:

In order to separate constant losses into iron and mechanical losses, No-load test should be conducted with variable voltage and rated frequency.

Conduct the experiment for different values of applied voltage and note down the watt meter reading and subtract the no load stator copper losses from that reading then constant losses will be obtained.

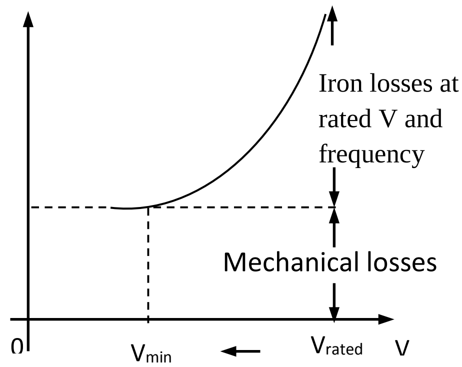
Iron losses $\propto V_1^2$ (for constant frequency of operation)

$\downarrow$ Iron losses $\propto V_1^2 \downarrow$

But mechanical losses almost constant

(Speed is constant as, frequency is constant)

Constant $\downarrow$ T $\propto \frac{sV_1^2}{R_2} \downarrow$ constant; s is constant



- By keeping frequency constant if applied voltage of induction motor reduced under no-load condition the speed of the induction motor is always constant. Therefore mechanical losses which depend on constant are almost constant.
- By keeping frequency constant if applied voltage of induction motor is reduced under load condition the speed of induction motor falls to maintain the load torque constant that's why the mechanical losses the induction motor will decrease.

  Once applied voltage is virtually zero iron losses disappears zero.

  From graph we measure mechanical losses
- By keeping applied voltage constant if frequency of operation is varied then mechanical losses will varies irrespective of whether the machine is loaded or not.

Deviation in No-load test if it is conducting at rated frequency but at less than rated voltage:

f = Rated frequency: V_1 < Rated voltage

1. $\downarrow \phi_R \alpha \frac{V_1 \downarrow}{f}$ (Air gap flux is less than the rated flux)

2. $I_\mu \downarrow$

3. $\downarrow$ Iron losses $\alpha\ V_1^2 \downarrow$

4. Mechanical losses are constant.

5. Constant losses are reduces.

6. $I_{wm} \downarrow$

7. $\downarrow I_0 = \sqrt{I_\mu{}^2 + I_{wm}{}^2}$

8. No load Stator cu losses $= 3I_0^2 R_1 \downarrow$

9. So no load power at reduced voltage rated frequency< no load power at rated frequency and voltage

10. Wattmeter reading $\downarrow$

11. $\cos\phi_0 \uparrow\ \because I_\mu \downarrow$

12. T_{st} and T_{max} are proportional to square of voltage and they will decrease.

Deviation in No-load test if it is conducting at rated voltage but at less than rated frequency:

f < Rated frequency; V_1 = Rated voltage

1. $\uparrow \phi_R \alpha \frac{V_1}{f \downarrow}$ (Air gap flux is more than the rated flux)

2. $I_\mu \uparrow$

3. $\uparrow$ Iron losses α ($w_h \alpha \frac{V^{1.6}}{f^{0.6}}$ and $w_e \alpha\ V_1^2$)

4. Mechanical losses are reduces.

5. Constant losses almost maintain constant (from 3 and 4)

6. I_{wm} = Constant

7. $\uparrow I_0 = \sqrt{I_\mu{}^2 + I_{wm}{}^2}$

8. No load Stator cu losses $= 3I_0^2 R_1 \uparrow$

9. So no load power at rated voltage reduced frequency< no load power at rated voltage and frequency

10. Wattmeter reading $\uparrow$

11. $\cos\phi_0 \downarrow\ \because I_\mu \uparrow$

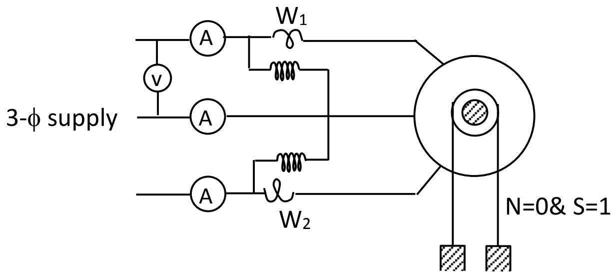
12. Both T_{st} and T_{max} will increase.

BLOCKED ROTOR TEST (or) LOCKED ROTOR TEST:

Objective:

- To find out the full load copper losses in induction motor
- To find out total resistance and reactance of induction motor when referred to stator side.

Circuit Connection

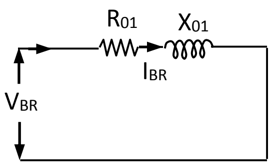


Procedure

By applying rated voltage under blocked rotor test the motor will take very high current and this current is short circuit current and is 5 to 8 times full load current. So we have to apply reduced voltage initially and increase the voltage up to the current drawn by the motor is rated current (It is mentioned on nameplate].

Observation:

The equivalent circuit of machine in blocked rotor test is:



A 3Ø induction motor under blocked rotor condition is equal to a transformer with short circuited secondary. That's why blocked rotor test on induction motor is equivalent to short circuit test

Calculations

Short circuit current (I_{SC}):

This is the amount of current that would flow through stator winding under blocked rotor test corresponding to rated applied voltage. High current is called short circuit current I_{sc}

Blocked rotor current (I_{BR}):

This is the current that will flow through stator winding which is equal to rated current of motor under blocked rotor condition corresponding to reduced applied voltage.

Readings are:

V_{BR} = Line voltage
(10 to 12% of rated voltage)

$I_{BR} = I_{SC}$ Line current
(Blocked rotor current)

$W_{BR} = W_1+W_2$
= losses in Induction motor under blocked rotor condition
= F. L cu losses + Small amount of Iron loss (αV_{BR}).

$\Rightarrow W_{BR} \approx$ F.L cu losses

Under assumptions that small amount of Iron losses corresponding 'V_{BR}' is neglected. The wattmeter reading during blocked rotor test can be approximately taken as full load copper losses.

$W_{BR} = 3I_{BR}^2 R_{01}/ph$

$$R_{01}/ph = \frac{W_{BR}}{3\times I_{BR}^2/ph}$$

$$V_{BR}/ph = I_{BR}Z_{01}/ph$$

$$Z_{01}/ph = \frac{V_{BR}/ph}{I_{BR}/ph} \text{ and } X_{01}/ph = \sqrt{(Z_{01}/ph)^2 - (R_{01}/ph)^2}$$

Name plate details of induction machine:

3ϕ, Δ, 6pole, 440V,50Hz,100A, 5HP,950 r.p.m

Full load speed
Net mechanical power output at full load condition.
Full load stator current
Rated frequency
Stator terminal voltage
Poles of each phase of stator
Type of connection
Stator phase

1 H.P = 735. 5 W

Circle diagram is useful for getting net mechanical power output at different loads like full load, half full load and $\frac{1}{4}$ full load, name plate details doesn't useful to get net mechanical power output.

Name plate details of induction machine:

3–ϕ, Δ 6 pole, 440V, 50Hz, 100A, 5HP

Mechanical power input at full load.

Full load stator current

Full load terminal voltage

Efficiency:

$$\text{Efficiency } \eta = \frac{\text{output power}}{\text{input power}}$$

$$\eta_{\text{motor at full load}} = \frac{\text{Net mechanical power output at F.L}}{\text{Electrical power input at F.L}}$$

Mentioned on name plate

$$\Rightarrow \frac{\text{Net mechanical power output at F.L}}{\text{Net mechanical power output at F.L} + \text{constant \& varible losses}}$$

Obtained from, No-load test

Obtained from; Blocked rotor test

$$\eta_{\text{generator at full load}} = \frac{\text{Net Electrical power output at full load}}{\text{Mechanic power input at full load}}$$

$$\Rightarrow \frac{\text{Net mechanical power input at full load} - \text{Variable losses} - \text{Constant losses}}{\text{Net mechanical power input at full load}}$$

A 12 pole,3 –phase ,50 Hz induction motor draws 280A and 110kW under the blocked rotor test. Find the started torque when switched on direct to rated voltage and frequency supply. Assume the stator and rotor copper losses to be equal under the blocked rotor rest.

(a) $r_1 = r_2' = 0.234$ *ohms; starting torque=1051Nm*

A 110 V, 3-phase, star connected induction motor takes 25 A at a line voltage of 30V with rotor locked. With this voltage, power input to the motor is 440W and core loss is 40 W. The dc resistance between pair of stator terminals is 0.1 ohms. If the ration of a.c to d,c resistance is 1.6. Find the equivalent leakage reactance/phase of the motor and the stator and rotor resistance per phase.